(Pages : 4) **2304**

Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, May 2013
(2008 Scheme)
Branch : Electronics and Communication
08.401 ENGINEERING MATHEMATICS III – PROBABILITY AND RANDOM PROCESSES (TA)

Time: 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions of Part – **A** and one **full** question **each** from Module – **I**, Module – **II** and Module – **III** of Part – **B**.*

PART – A

1. The probability function of an infinite discrete distribution is given by $P[X = x] = \frac{1}{2x}$ $(x = 1, 2, \ldots \infty)$ verify that the total probability is 1 and also find the mean and variance of the distribution.

2. The probability density function of a random variable X is

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x < 2 \\ 0 & x \ge 2 \end{cases}$$ Find the cumulative distribution function of X.

3. A normal distribution has mean $\mu = 20$ and standard deviation $\sigma = 10$.

   Find $P(15 \le X \le 40)$.

4. The joint probability density function of a two-dimensional random variable (X, Y) is

$$f(x, y) = \begin{cases} \frac{8}{9}xy, & 1 < x < y < z \\ 0, & \text{otherwise} \end{cases}$$

   i) Find the marginal density function of X.

   ii) Find the conditional density function of Y given X = x.

P.T.O.

5. Show that the linear correlation coefficient lies between −1 and 1.
6. Define SSS process and WSS process. What is the difference between them ?
7. Find the mean and variance of a Poisson process.
8. Given that the autocorrelation function for a stationary ergodic process with no periodic components is $R(\tau) = 25 + \frac{4}{1+6\tau^2}$

   Find the mean value and variance of the process {X(t)}.
9. State Wiener – Khinchin Theorem. If the autocorrelation function $R(\tau) = 1$ find the spectral density S(w).
10. Let $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ be a stochastic matrix. Examine whether it is regular.

(10×4= 40 Marks)

PART – B

Module – I

11. a) Out of 800 families with 4 children each, how many families would be expected to have
    i) 2 boys and 2 girls
    ii) atmost 2 girls
    iii) children of both sexes.

    b) If X has a distribution with probability density function $f(x) = e^{-x}, 0 \le x < \infty$. Use Chebychev's inequality to obtain the lower bound to the probability $P(-1 \le x \le 3)$ and compare it with the actual value.

    c) The joint probability density function of two random variables X and Y is given by $f(x, y) = kxy\ e^{-(x^2+y^2)}, x > 0, y > 0$.

    Find the value of k and also prove that X and Y are independent. **(7+7+6=20)**

12. a) An electrical firm manufactures light bulbs that have a life, before burn out, that is normally distributed with mean equal to 800 hrs and a standard deviation of 40 hrs. Find the probability a bulb burns
    i) more than 834 hrs
    ii) less than 900 hrs
    iii) between 778 and 834 hrs.

b) Find the coefficient of correlation between X and Y from the following data

X :	10	14	18	22	26	30
Y :	18	12	24	16	30	36

c) The joint probability distribution two random variables X and Y is given by

X \ Y	0	1	2
0	0.1	0.04	0.06
1	0.2	0.08	0.12
2	0.2	0.08	0.12

Examine whether X and Y are independent ? **(7+7+6=20)**

Module – II

13. a) Show that the random process

$X(t) = A \cos \lambda t + B \sin \lambda t$, where A and B are random variables is a wide-sense stationary if

i) $E(A) = E(B) = 0$ ii) $E(A^2) = E(B^2)$ and iii) $E(AB) = 0$.

b) If $\{X(t)\}$ is a WSS process with autocorrelation function $R(\tau) = Ae^{-\alpha|\tau|}$ determine the second order moment of the random variable $X(8) - X(5)$.

c) On the average, a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of ships sighted in a given length of time is a Poisson variate; find the probability of sighting

i) 6 ships in the next half an hour

ii) at least one ship in the next 15 minutes . **(7+7+6=20)**

14. a) Show that the random process $X(t) = A \cos(\omega_0 t + \theta)$ is a wide-sense stationary where A and ω_0 are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.

b) Two random process $\{X(t)\}$ and $\{Y(t)\}$ are defined by $X(t) = A \cos \omega t + B \sin \omega t$ and $Y(t) = B \cos \omega t - A \sin \omega t$. Show that X(t) and Y(t) are jointly wide-sense stationary, if $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$; $E(AB) = 0$ and ω is a constant.

c) A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 5 minutes period. **(7+7+6=20)**

Module – III

15. a) If $\{X(t)\}$ is a random signal process with $E[X(t)] = 0$ and $R(\tau) = e^{-2\lambda|\tau|}$. Find the mean and variance of the time average of $\{X(t)\}$ over $(-T, T)$. Is it mean-ergodic ?

b) Consider a Markov chain with state space $\{0, 1, 2\}$ and the transition probability matrix is $P = \begin{bmatrix} 0 & 1 & 0 \\ 1-p & 0 & p \\ 0 & 1 & 0 \end{bmatrix}$

i) Find P^2 and show that $P^2 = P^4$ ii) Find P^n, $n \geq 1$

c) Find the power spectral density of a WSS process with autocorrelation function $R(\tau) = e^{-\alpha\tau^2}, \alpha > 0$. **(7+7+6=20)**

16. a) Define irreducible Markov chain. Show that the matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

is the transition probability matrix of an irreducible Markov chain ?

b) If $\{X(t)\}$ is a WSS process with $E[X(t)] = 2$ and $R(\tau) = 4 + e^{-|\tau|/10}$

Find the mean and variance of $S = \int_0^1 X(t)dt$.

c) Find the power spectral density of the random binary transmission process whose autocorrelation function is

$$R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} & \text{for } |\tau| \leq T \\ 0 & \text{elsewhere} \end{cases}$$

(7+7+6=20)
